A Spatial Interaction Model With Spatially Structured Origin and Destination Effects

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Abstract

We introduce a Bayesian hierarchical regression model that extends the traditional least-squares regression model used to estimate gravity or spatial interaction relations involving origin-destination flows. Spatial interaction models attempt to explain variation in flows from n origin regions to n destination regions resulting in a sample of $N = n^2$ observations that reflect an n by n flow matrix converted to a vector. Explanatory variables typically include origin and destination characteristics as well as distance between each region and all other regions. Our extension introduces latent spatial effects parameters structured to follow a spatial autoregressive process. Individual effects parameters are included in the model to reflect latent or unobservable influences at work that are unique to each region treated as an origin and destination. That is, we estimate $2n$ individual effects parameters using the sample of $N = n^2$ observations. We illustrate the method using a sample of commodity flows between 18 Spanish regions during the 2002 period.

KEYWORDS: commodity flows, spatial autoregressive random effects, Bayesian hierarchical models, spatial connectivity of origin-destination flows.

1 Introduction

We introduce a Bayesian hierarchical regression model that includes latent spatial effects structured to follow a spatial autoregressive process to investigate commodity flows between origin and destination regions. The sample data involves $n = 18$ Spanish regions where the commodity flows have been organized as an n by n "origin-destination (OD) flow matrix" that we label Y. Without loss of generality, the row elements of the matrix $Y_{ij}, i = 1, \ldots, n$ reflect the dollar value (in millions of Euros) of commodity flows originating in region j that were shipped to destination regions $i = 1, \ldots, n$. We therefore treat the columns as "origins" of the commodity flows and the rows are "destinations" of the flows.

The term 'spatial interaction models' has been used by Sen and Smith (1995) and others to label models that focus on flows between origins and destinations. These models seeks to explain variation in the level of flows across the sample of $N = n^2$ OD pairs by relying on a function of distance between the origin and destination regions as well as explanatory variables consisting of origin and destination characteristics of the regions. Conventional spatial interaction models assume that using distance as an explanatory variable will eradicate the spatial dependence among the sample OD pairs allowing use of least-squares estimation methods. We note that use of least-squares also requires a normal distribution for the dependent variable magnitudes contained in the vectorized flow matrix $y = vec(Y)$ for valid inference using traditional regression $t-$tests and measures of statistical significance. However, unbiased estimates for the slope parameters can still be obtained for non-normal disturbances and associated distribution for the y variable. We assume a normal distribution for the disturbances to be the case in developing our model which rules out use of our methodology in cases where the flow matrix is sparse containing a large number of zero entries reflecting a lack of interaction between regions. Sparse flow matrices typically arise when regions are defined using a fine spatial scale or a short time period over which observed flow information is collected.

A spatial autoregressive structure is use to structure two latent regional effects vectors, one for regions reflecting origins of the commodity flows and a second for the destination regions. The spatial autoregressive prior structure reflects a prior belief that the latent origin effects levels should be similar to those from regions that neighbor the region where commodity flows originate. A second regional effects vector imposes the same prior belief regarding the effects levels of destination regions and their neighbors. Intuitively, the missing covariates for the origin region that contribute to model heterogeneity may have a spatial character, so that the same missing covariates would influence nearby regions. The same intuition applies to the destination regions, missing covariates are likely to exert similar heterogeneity as those from neighboring destinations.

We use posterior estimates of the origin and destination latent effects to identify regions that exhibit positive and negative effects magnitudes. Since the effects parameters have a prior mean of zero, positive and negative posterior effects estimates can be interpreted as measuring the magnitude and influence of missing covariates or latent unobservable factors on the commodity flow process.

The hierarchical regression model utilizes recent work by Smith and LeSage (2004) that introduced Bayesian Markov Chain Monte Carlo (MCMC) estimation methods for these models where the regional effects parameters are modelled using data augmentation. There is a large literature on

Bayesian hierarchical spatial models (see Besag, York, and Mollie, 1991, Besag and Kooperberg, 1995, Cressie 1995 and Gelfand and Banerjee, 2004), that relies on the conditional autoregressive (CAR) spatial prior to structure the regional effects parameters. In contrast, our approach utilizes the spatial autoregressive process (SAR) from Smith and LeSage (2004) as a prior structure on the regional effects parameters.

Individual effects estimates are notoriously difficult to estimate with precision in conventional hierarchical linear models (Gelfand, Sahu and Carlin, 1995. Our approach to structuring two sets of regional/spatial effects parameters overcomes these problems in two ways.

First, the spatial autoregressive structure placed on the latent effects parameters for the origin depend on one hyperparameter measuring the strength of spatial dependence and another representing a scalar variance parameter. These two parameters are introduced in the context of a sample of $n^2 = N$ observations, where $n = 18$ regions and $N = 324$ represents the sample of origin-destination pairs that arise from vectorizing the origin-destination flow matrix. As noted above, the $N = n^2$ sample size arises from vectorizing an n by n origin-destination flow matrix Y, where the rows of the matrix Y reflect commodity flow destinations and the columns reflect regions where the commodity flows originate. We estimate only n latent regional "origin" effects parameters, one for each region treated as an origin, allowing us to rely on n sample data observations *for each* of the $i = 1, \ldots, n$ origin effect parameter estimates. In fact, since the n origin effects parameters are derived from the two hyperparameters that completely determine the spatial autoregressive process assigned to govern these effects, we could view this as relying on a sample of N observations to estimate two parameters. A similar situation holds for the case of the n

"destination" effects estimates for the destination regions. Again, we rely on the larger sample of N observations to produce estimates of the two parameters specifying the spatial autoregression assigned to these n effects parameters.

Second, the spatial autoregressive (SAR) structure assumed to govern the origin and destination effects introduces additional sample data information in the form of an n by n spatial contiguity matrix that describes the spatial connectivity structure of the sample regions. This additional spatial structure in conjunction with the spatial autoregressive process assumption provides a parsimonious parameterization of the regional effects parameters. This is in contrast to the typical assumption of a normal distribution with zero mean and constant scalar variance assigned as a prior for non-spatial latent effects parameters. Our approach of estimating two sets of n latent effects based on a sample of size $N = n^2$ also differs from the conventional approaches that estimate a latent effect parameter for all sample observations, which would be N in our case.

As noted above, use of a spatial autoregressive process as a prior for the latent effects parameters also differs from most hierarchical spatial linear models that rely on a conditional autoregressive (CAR) process prior, or an intrinsic conditional autoregressive (ICAR) process. In this regard, we follow Smith and LeSage (2004) who introduced spatially structured SAR priors for latent effects in the context of a probit model. There are numerous advantages to the SAR prior over the CAR. We will have more to say about this in Section 2 where details regarding the model specification are provided.

Section 2 of the paper describes the conventional spatial interaction model along with our extension of this model to include the latent origin and destination regional effects parameters. Section 3 applies the method to a data generated example where the true parameters are known in order to illustrate and assess the proposed methodology, and section 4 applies the method to a sample of commodity flows during the year 2002 between 18 Spanish regions.

2 Empirical modeling of commodity flows

In section 2.1 we review the traditional gravity or spatial interaction model that assumes the OD flows contained in the dependent variable vector $y = vec(Y)$ are independent, consistent with the Gauss-Markov assumptions for least-squares. Section 2.2 describes our extension to this model that introduces spatially structured regional effects parameters.

2.1 Conventional gravity models

A conventional gravity or spatial interaction model would rely on an n by k matrix of explanatory variables that we label X, containing k characteristics for each of the n regions and the flow matrix Y vectorized, so that each column of the matrix Y is stacked. Given the origin-destination format of the vector y, where observations 1 to n reflect flows from origin 1 to all n destinations, the matrix X would be repeated n times to produce an N by k matrix representing destination characteristics that we label X_d (see LeSage and Pace, 2008). We note that X_d equals $\iota_n \otimes X$, where ι_n is an n by 1 vector of ones. A second matrix can be formed to represent origin characteristics that we label X_o. This would repeat the characteristics of the first region n times to form the first n rows of X_o, the characteristics of the second region n times for the next n rows of X_o and so on, resulting in an N by k matrix that we label $X_o = X \otimes \iota_n$. The distance from each origin to destination is also included as an explanatory variable vector in the gravity model. We let G represent the n by n matrix of distances between origins and destinations, and thus $g = vec(G)$ is an N by 1 vector of these distances from each origin to each destination formed by stacking the columns of the origin-destination distance matrix into a variable vector.

This results in a regression model of the type shown in (1).[1]

$$y = \alpha\iota_N + X_d\beta_d + X_o\beta_o + \gamma g + \varepsilon \tag{1}$$

In (1), the explanatory variable matrices X_d, X_o represent N by k matrices containing destination and origin characteristics respectively and the associated k by 1 parameter vectors are β_d and β_o. The scalar parameter γ reflects the effect of distance g, and α denotes the the constant term parameter. The N by 1 vector ε represent disturbances and we assume $\varepsilon \sim N[0, \sigma^2 I_N]$, where we use $N_k[\mu, \Sigma]$ to represent a $k-$variate normal distribution with mean μ and variance-covariance Σ.

One problem encountered in modeling the n by n matrix of flows, which we designate using Y is that main diagonal elements reflect intraregional flows and are typically large relative to the off-diagonal elements that represent interregional flows. LeSage and Pace (2008) suggest creating a separate model for intraregional flows from the main diagonal of the flow matrix. They do this by setting all elements of the covariate matrices X_d, X_o corresponding to the main diagonal of the flow matrix to zero, and moving these elements to a new N by k matrix which we label X_i. The matrix X_i has zeros except for those elements associated with the main diagonal of the

[1] If one starts with the standard gravity model and applies a log-transformation, the resulting structural model takes the form of (1) (c.f., equation (6.4) in Sen and Smith, 1995).

flow matrix. This which prevents the variables in X_d, X_o from entering the interregional flow model, creating a separate set of explanatory variables to explain this variation in the matrix X_i. In our applied illustration we use a vector for the matrix X_i containing the main diagonal flow elements from a previous time period. In many circumstances previous period flows may not be available, necessitating the approach of LeSage and Pace (2008). We note that one need not employ all variables in the matrix X, since a subset of these might work well to explain intraregional flows. For example, the area of a region, the income level and population might work well to explain the magnitude of flows within the region. It is typically the case that intraregional flows are considered a nuisance in these models, since the focus is on explaining variation in interregional flows. Introduction of the separate models for inter- and intraregional flows allows the parameter estimates for β_d, β_o to better reflect the impact of origin and destination characteristics on the interregional flow levels. The conventional gravity model approach allows the large main diagonal flow elements to influence these parameter estimates. A frequent practice in applied modeling is to set the diagonal elements of the flow matrix to zero (see Tiefelsdorf, 2003 and Fischer et al., 2006). In a spatial context where neighboring regions are not independent, setting these elements to zero will exert an impact on the pattern of spatial dependence.

A second problem that arises is the need to store sample data information in the N by k matrices, X_d, X_o, X_i, which can consume a large amount of computer memory when n is large. For example, a model involving 3,000 US county-level flows would require three 9 million by k matrices. We extend the moment matrix approach of LeSage and Pace (2008) to our model. They point out that rather than work with matrices $X_d = \iota_n \otimes X, X_o = X \otimes \iota_n$, it is possible to work with smaller k by k matrices $X'X$.

This can be accomplished for the least-squares model by letting:

$$y = Z\delta + \varepsilon \tag{2}$$

where: $Z = [\,\iota_N \quad \tilde{X}_d \quad \tilde{X}_o \quad X_i \quad g\,]$, and $\delta = [\,\alpha \quad \beta_d \quad \beta_o \quad \beta_i \quad \gamma\,]'$.

We assume the matrix X is in deviation from means form, and define the n by n matrix G to contain the interregional distances (in deviation from means form). Based on the introduction of the matrix X_i described above, the matrices: $\tilde{X}_d = X_d - X_i$, $\tilde{X}_o = X_o - X_i$, where $X_d = I_n \otimes X$, $X_o = X \otimes I_n$. The resulting moment matrices take the form:

$$Z'Z = \begin{bmatrix} N & 0 & 0 & 0 & 0 \\ 0 & (n-1)X'X & -X'X & 0 & X'\mathrm{d}(G) - X'G\iota_n \\ 0 & -X'X & (n-1)X'X & 0 & X'\mathrm{d}(G) - X'G\iota_n \\ 0 & 0 & 0 & X'X & X'\mathrm{d}(G) \\ 0 & \iota_n' G'X - \mathrm{d}(G')X & \iota_n' G'X - \mathrm{d}(G')X & \mathrm{d}(G')X & tr\left(G^2\right) \end{bmatrix}$$

Where $\mathrm{d}(G)$ is an n by 1 vector containing the diagonal elements of the matrix G. For the case of the $Z'y$ we have that: $(X_d - X_i)'y = (I_n \otimes X)'y - X_i'y = X'Y\iota_n - X'\mathrm{d}(Y)$ and $(X_o - X_i)'y = X_o'y - X_i'y = X'Y'\iota_n - X'\mathrm{d}(Y)$, yielding:

$$\begin{aligned} Z'y &= [\,\iota_n' Y \iota_n \quad (X_d - X_i)'y \quad (X_o - X_i)'y \quad X_i'y \quad tr(GY)\,]' \\ &= [\,\iota_n' Y \iota_n \quad X'Y\iota_n - X'\mathrm{d}(Y) \quad X'Y'\iota_n - X'\mathrm{d}(Y) \quad X'\mathrm{d}(Y) \quad tr(GY)\,] \end{aligned}$$

Least-squares estimates for the model can now be produced using: $(Z'Z)^{-1}Z'y$, which involves inversion of the $3k+2$ by $3k+2$ matrix $Z'Z$.

2.2 A Bayesian hierarchical gravity model

LeSage and Pace (2008) point to the implausible nature of the assumption that OD flows contained in the dependent variable vector y exhibit no spatial dependence. They note that the gravity model makes an attempt at modeling spatial dependence between observations using distance, but if each region exerts an influence on its neighbors this might be inadequate. For example, neighboring origins and destinations may exhibit estimation errors of similar magnitude if underlying latent or unobserved forces are at work or missing covariates exert a similar impact on neighboring observations. They point out that agents located at origins nearby in space may experience similar transport costs and profit opportunities when evaluating alternative destinations.

We extend the model from (1) by introducing two n by 1 vectors of regional effects parameters, one for each region treated as an origin θ and another for destination regions ϕ. This model can be expressed as:

$$\begin{aligned} y_j &= z_j\delta + v_j\theta + w_j\phi + \varepsilon_j \quad\quad \varepsilon \sim N_N[0, \sigma_\varepsilon^2 I_N] && (3) \\ y &= Z\delta + V\theta + W\phi + \varepsilon \\ \theta &= \rho_o D\theta + u_o, \quad\quad u_o \sim N_n[0, \sigma_o^2 I_n] && (4) \\ \phi &= \rho_d D\phi + u_d \quad\quad u_d \sim N_n[0, \sigma_d^2 I_n] && (5) \end{aligned}$$

Where $z_j = [1 \quad \tilde{x}_{j,d} \quad \tilde{x}_{j,o} \quad x_{j,i} \quad g_j]$, with $\tilde{x}_{j,d}, \tilde{x}_{j,o}, x_{j,i}$ representing row elements from the corresponding matrices: $\tilde{X}_d, \tilde{X}_o, X_i$. The vector, $v_j =$

$(v_{j1}, \ldots, v_{jn})$ identifies region j as an origin region and $w_j = (w_{j1}, \ldots, w_{jn})$ identifies destination regions. Given our configuration for the commodity flow matrix with columns as origins and rows as destinations, the matrices $W = I_n \otimes \iota_n$ and $V = \iota_n \otimes I_n$ such that v_j and w_j represent the jth row of these mutually exclusive N by n matrices.

For the spatial effects parameters we rely on the spatial autoregressive priors shown in (4) and (5), where D is an n by n row-normalized first-order spatial contiguity matrix. This matrix reflects the spatial configuration of the regions in terms of common borders, with row-sums of unity by virtue of the row-normalization.

We provide an economic motivation for inclusion of the spatial effects vectors θ and ϕ in the model in the sequel. Bonacich (1987) introduced a centrality measure in the context of social networking that has come to be known as the Bonacich centrality index. In our context, we can view the spatial weight matrix D as an n by n adjacency matrix related to the spatial configuration or network of our observations/regions. The Bonacich index for region i counts the total number of paths in the network defined by our spatial contiguity weight matrix D that start at region i. These consist of the sum of all loops from region i to itself, and the sum of all outer paths from region i to every other region $j \neq i$. Specifically, Bonacich defines a vector: $(I_n - \rho_d D)^{-1}\iota_n = (\sum_{k=0}^{\infty} \rho_d^k D^k)\iota_n$, that sums up the elements of $(I_n - \rho_o D)^{-1}$. The i,jth element represents a count of the number of paths in our regional configuration that start at region i and end at j, with paths of length k weighted by the parameter ρ^k.[2] We note that the SAR structure placed on the origin (destination) effects parameters reflect

[2]For our conventional spatial contiguity matrix D which has zeros on the diagonal and row-sums of unity, the inverse is well defined for $\rho < 1$.

a weighted variant of the Bonacich network centrality measure: $\theta = (I_n - \rho_o D)^{-1} u_o$, where the heterogenous vector u_o replaces the homogenous ι_n vector.[3] The centrality measure indicates regions with more/less contiguous neighbors, and therefore posits large/smaller effects estimates for regions in accordance with this. For example, regions on the edge of Spain as well as (physically) large regions would have less contiguous neighbors, and we would expect to see smaller (in absolute value terms) effects estimates. This is in fact consistent with our empirical findings, an issue taken up when presenting a map of the effects estimates.

Abstracting from any economic motivation, when $\rho_o > 0$,the SAR prior structure leads to larger origin and destination effects parameters θ associated with regions that exhibit greater "network centrality,", when regions are viewed as origins or destinations of commodity flows.

An economic motivation for the vectors θ, ϕ on which we place the SAR prior structure is provided by a result from Ballester, Calvó, and Zenou (2006). They show that in a noncooperative network game with linear quadratic payoffs as a return to effort, the Nash equilibrium effort exerted by each player is proportional to the Bonacich centrality of the player's situation in the network. The game involves efforts that exhibit local complementarity with efforts of other players.

Drawing on their result, we can posit the existence of unobservable inputs, that play the role of effort exerted in Ballester, Calvó, and Zenou (2006). These inputs can be related to intra- and interregional commodity flows using a strictly concave bilinear payoff function. The unobservable inputs are not reflected in the regional characteristics measured by X_d, X_o, X_i or the distances g, on which the model is already conditioned. Ballester,

[3] Of course for the destination effects parameters we have: $\phi = (I_n - \rho_d D)^{-1} u_d$.

Calvó, and Zenou (2006) show that for the case of a simultaneous move n player game there is a unique (interior) Nash equilibrium (see Theorem 1, Remark 1, Ballester, Calvó, and Zenou, 2006) for effort exerted. The equilibrium (effort, or in our case unobservable input usage) is proportional to our weighted variant of the Bonacich network centrality measure. They establish that this result applies to both symmetric and asymmetric structures of complementarity across the regions represented by the n by n matrix D, and the matrix D is required to obey the usual spatial autoregressive process restrictions.[4]

This result motivates that when we model a cross-section of observed commodity flows at a particular point in time, after conditioning on observable regional factors in the explanatory variable matrices X_d, X_o, X_i and g, unobservable factors are likely to exhibit the SAR structure we use as a prior for the vectors θ and ϕ

Turning to specification of the remaining priors for parameters in our model, we assign an uninformative inverse-gamma (IG) prior for the parameters σ_o^2, σ_d^2 and σ_ε^2, taking the form:

$$\pi(\sigma_o^2), \pi(\sigma_d^2), \pi(\sigma_\varepsilon^2) \sim IG(\nu_1, \nu_2) \tag{6}$$

Where in the absence of prior information, it seems reasonable to rely on the same prior for σ_o^2, σ_d^2 and σ_ε^2. It also seems reasonable to assign values $\nu_1 = 2, \nu_2 = 1$ which reflects an uninformative prior with mean = 1, mode = 0.33, and infinite variance.

The spatial dependence parameters are known to lie in the stationary interval: $[\kappa_{\max}^{-1}, \kappa_{\min}^{-1}]$ where $\kappa_{\min} < 0, \kappa_{\max} > 0$ denote the minimum and

[4]Row-sums of unity and zeros on the main diagonal.

maximum eigenvalues of the matrix D, (see for example, Lemma 2 in Sun et al., 1999). We rely on a uniform distribution over this interval as our prior for ρ_o, ρ_d, that is:

$$\pi(\rho_o), \pi(\rho_d) \sim U[\kappa_{\max}^{-1}, \kappa_{\min}^{-1}] \propto 1 \tag{7}$$

Solving for θ and ϕ in terms of u_o and u_d suggests a normal prior for the origin and destination spatial effects vectors taking the form:

$$\begin{aligned}
\theta|\rho_o, \sigma_o^2 &\sim N_n[0, \sigma_o^2(B_o'B_o)^{-1}] \\
\phi|\rho_d, \sigma_d^2 &\sim N_n[0, \sigma_d^2(B_d'B_d)^{-1}] \\
B_o &= (I_n - \rho_o D) \\
B_d &= (I_n - \rho_d D)
\end{aligned}$$

We note that B_o, B_d are non-singular for conventional row-normalized first-order spatial contiguity matrices D and the spatial dependence parameters ρ_o, ρ_d in the interval: $[\kappa_{\max}^{-1}, \kappa_{\min}^{-1}]$. This leads to a proper prior distribution in contrast to the well-known intrinsic CAR prior introduced by Besag and Kooperberg (1995).

We also point out that when the parameters $\rho_o = \rho_d = 0$, our model collapses to the special case of a normal prior for the random effects vectors with means of zero for both effects and constant scalar variances σ_o^2 and σ_d^2, so our SAR prior specification subsumes this as a special case. It should be noted that estimates for these two sets of random effects parameters are identified, since a set of n mutually exclusive sample data observations are aggregated through the vectors v_i and w_i to produce each estimate θ_i, ϕ_i in the vector of parameters θ and ϕ.

Finally, we use a normal prior distribution for the parameters $\delta = [\,\alpha \quad \beta_o \quad \beta_d \quad \beta_i \quad \gamma\,]'$ associated with the covariates in the explanatory variables matrix $X = [\,\iota \quad X_d \quad X_o \quad X_i \quad g\,]$ centered on zero with a large standard deviation:

$$\pi(\delta|\psi) \sim N_{3k+2}[0, T] \tag{8}$$

Where $3k + 2$ denotes the number of explanatory variables in the matrix Z, $T = \omega^2 I_{3k+2}$, with $\omega^2 = 1,000$.

2.3 Related spatial effects models

In place of the SAR prior, we could rely on variants of the CAR prior that result in proper priors, Sun et al. (2000) among others. Other examples of the SAR prior in the context of random spatial effects are Smith and LeSage (2004), and LeSage, Fischer and Scherngell (2007).

Banerjee, Gelfand and Polasek (2000) model the timeliness of postal service flows using a binary response indicator for on-time or delayed mail as the dependent variable in place or our flows. Their model replaces the origin and destination effects terms: $v_j\theta$ and $w_j\phi$ with dummy variables (fixed effects) and associated parameter estimates. They then proceed to model the disturbances ε_j using a spatial process model. This type of approach focuses on disturbance heterogeneity and covariance which can be modeled in an effort to improve the precision of the estimates.

We note the contrast with our model where introduction of the spatially structured effects parameters will have a direct impact on the resulting estimates $\beta_d, \beta_o, \beta_i$ and γ. To see this consider the conditional distribution of

δ:

$$\delta|\theta, \phi, \rho_d, \rho_o, \sigma_u \sigma_o, \sigma_\varepsilon = (Z'Z)^{-1}Z'(y - V\theta - W\phi) \tag{9}$$
$$= (Z'Z)^{-1}Z'[y - V(I_n - \rho_d D)^{-1}u_d - W(I_n - \rho_o D)^{-1}u_o]$$

in contrast to that from a model where V and W represent fixed effects, and θ, ϕ the associated parameters, with the disturbances modeled by a spatial process whose variance-covariance structure we represent by Ω:

$$\tilde{\delta}|\theta, \phi, \rho_d, \rho_o, \sigma_u \sigma_o, \sigma_\varepsilon = (Z'\Omega Z)^{-1}Z'\Omega(y - V\theta - W\phi) \tag{10}$$

In the fixed effects model (10), $E(\tilde{\delta}) = (Z'Z)^{-1}Z'(y - V\theta - W\phi)$, so the spatial process model for the disturbances has no impact on the parameter estimates $\tilde{\delta}$. If we eliminated the spatial autoregressive priors in (12) and (13) and estimated coefficient vectors θ, ϕ for the model: $y = Z\delta + V\theta + W\phi + \varepsilon$, we would have a fixed effects model.[5]

In contrast, the model containing SAR structured effects results in destination or origin specific shocks, (u_d, u_o), exerting an influence on the parameter estimates. The amount of influence is described by the weighted Bonacich centrality measures: $(I_n - \rho_d D)^{-1}u_d$ and $(I_n - \rho_o D)^{-1}u_o$.

Intuitively, the existence of spatially clustered unobserved latent influences should lead to an adjustment in the response of commodity flows (y) to destination and origin region characteristics (X_d, X_o) as well as distance

[5] Of course, one of the regions would need be eliminated from each of the matrices V, W to avoid have a perfect linear combination of dummy variables.

(g) and the intraregional model variables (X_i), captured in the parameters δ associated with each of these explanatory variables. Further, the magnitude of adjustment will depend on the weighted Bonacich centrality of the region where unobserved latent influences are operating.

2.4 MCMC estimation of the model

For notational convenience in the following discussion we restate the observation-level expression (3) of our model in matrix form:

$$
\begin{aligned}
y &= Z\delta + V\theta + W\phi + \varepsilon && (11)\\
\theta &= \rho_o D\theta + u_o \\
\phi &= \rho_d D\phi + u_d \\
\pi(\theta|\rho_o, \sigma_o^2) &\sim (\sigma_o^2)^{n/2}|B_o|\exp\left(-\frac{1}{2\sigma_o^2}\theta' B_o' B_o \theta\right) && (12)\\
\pi(\phi|\rho_d, \sigma_d^2) &\sim (\sigma_d^2)^{n/2}|B_d|\exp\left(-\frac{1}{2\sigma_d^2}\phi' B_d' B_d \phi\right) && (13)
\end{aligned}
$$

Where the expressions (12) and (13) reflect the implied prior for the spatial effects vector θ conditional on ρ_o, σ_o^2 and that for ϕ conditional on ρ_d, σ_d^2.

We use the normal linear model from (11) as the starting point to introduce the conditional posterior distributions that form the basis of an MCMC estimation scheme for our model. The basic scheme involves the following steps.

1. sample the regression parameters δ given $\theta, \phi, \rho_o, \rho_d, \sigma_o^2, \sigma_d^2, \sigma_\varepsilon^2$.

2. sample the noise variance σ_ε^2 given $\delta, \theta, \phi, \rho_o, \rho_d, \sigma_o^2, \sigma_d^2$.

3. sample the regional effects parameters θ, ϕ given $\rho_o, \rho_d, \sigma_o^2, \sigma_d^2, \delta, \sigma_\varepsilon^2$.

4. sample the spatial dependence parameters ρ_o, ρ_d given σ_o^2, σ_d^2.

5. sample the spatial effects variances σ_o^2, σ_d^2 given ρ_o, ρ_d.

Given the assumed prior independence of $\delta, \rho_o, \rho_d, \sigma_o^2, \sigma_d^2, \sigma_\varepsilon^2$, we have a joint posterior density for δ shown in (14).

$$
\begin{aligned}
p(\delta|\theta, \phi, \rho_o, \rho_d, \sigma_\varepsilon^2) &\propto \pi(\delta) \\
&\cdot \exp\{-\frac{1}{2\sigma_\varepsilon^2}(y - Z\delta - V\theta - W\phi)'(y - Z\delta - V\theta - W\phi)\} \\
&\propto \exp\{-\frac{1}{2\sigma_\varepsilon^2}(y - Z\delta - V\theta - W\phi)'(y - Z\delta - V\theta - W\phi)\} \\
&\cdot \exp\{-\frac{1}{2}\delta' T^{-1}\delta\} \qquad (14)
\end{aligned}
$$

In Appendix A, we show that this results in a multivariate normal conditional posterior distribution for δ taking the form shown in (15).

$$
\begin{aligned}
\delta|\theta, \phi, \rho_o, \rho_d, \sigma_o^2, \sigma_d^2, \sigma_\varepsilon^2, y, Z &\sim N_k[\Sigma_\delta^{-1}\mu_\delta, \Sigma_\delta^{-1}] \\
\mu_\delta &= \sigma_\varepsilon^{-2} Z'(y - V\theta - W\phi) \\
\Sigma_\delta &= (\sigma_\varepsilon^{-2} Z'Z + T^{-1}) \qquad (15)
\end{aligned}
$$

As already noted when discussing the least-squares variant of the model, it is not computationally efficient to work with the n^2 by $3k + 2$ matrix Z which involves repeating the smaller n by k sample data information matrix X through the use of kronecker product. We can rely on a similar moment matrix approach as set forth for the case of least-squares.[6]

[6]For clarity of presentation, we set forth conditional distributions involved in our sampling scheme in vector-matrix notation rather than the moment matrix form.

Taking a similar approach to that for δ, we have a joint posterior density for θ of the form:

$$
\begin{aligned}
p(\theta|\delta,\phi,\rho_o,\rho_d,\sigma_\varepsilon^2) &\propto \pi(\theta|\rho_o,\sigma_o^2) \\
&\cdot \exp\{-\frac{1}{2\sigma_\varepsilon^2}[V\theta-(y-Z\delta-W\phi)]'[V\theta-(y-Z\delta-W\phi)]\} \\
&\propto \exp\{-\frac{1}{2\sigma_\varepsilon^2}[V\theta-(y-X\delta-W\phi)]'[V\theta-(y-Z\delta-W\phi)]'\} \\
&\cdot \exp\{-\frac{1}{2\sigma_o^2}\theta' B_o' B_o \theta\}
\end{aligned}
$$

Which we show in Appendix A leads to a multivariate normal as the conditional posterior distribution for θ:

$$
\begin{aligned}
\theta|\delta,\phi,\rho_o,\rho_d,\sigma_o^2,\sigma_d^2,\sigma_\varepsilon^2,y,Z &\sim N_n[\Sigma_\theta^{-1}\mu_\theta,\Sigma_\theta^{-1}] \\
\mu_\theta &= \sigma_\varepsilon^{-2}V'(y-Z\delta-W\phi) \\
\Sigma_\theta &= (\frac{1}{\sigma_o^2}B_o'B_o+\frac{1}{\sigma_\varepsilon^2}V'V) \qquad (16)
\end{aligned}
$$

Similarly for the spatial effects vector ϕ we have:

$$
\begin{aligned}
\phi|\delta,\theta,\rho_o,\rho_d,\sigma_o^2,\sigma_d^2,\sigma_\varepsilon^2,y,Z &\sim N_n[\Sigma_\phi^{-1}\mu_\phi,\Sigma_\phi^{-1}] \\
\mu_\phi &= \sigma_\varepsilon^{-2}W'(y-Z\delta-V\theta) \\
\Sigma_\phi &= (\frac{1}{\sigma_d^2}B_d'B_d+\frac{1}{\sigma_\varepsilon^2}W'W) \qquad (17)
\end{aligned}
$$

The joint posterior distributions for ρ_o,ρ_d take the forms:

$$
\begin{aligned}
p(\rho_o|\delta,\theta,\phi,\sigma_o^2,\sigma_d^2,\rho_d,\sigma_\varepsilon^2,y) &\propto \pi(\theta|\rho_o,\sigma_o^2)\pi(\rho_o) \\
&\propto |B_o|\exp\left(-\frac{1}{2\sigma_o^2}\theta' B_o' B_o \theta\right) \\
p(\rho_d|\delta,\theta,\phi,\sigma_o^2,\sigma_d^2,\rho_o,\sigma_\varepsilon^2,y) &\propto \pi(\phi|\rho_d,\sigma_d^2)\pi(\rho_d) \\
&\propto |B_d|\exp\left(-\frac{1}{2\sigma_d^2}\phi' B_d' B_d \phi\right) \qquad (18)
\end{aligned}
$$

Which as noted in Smith and LeSage (2004) are not reducible to a standard distribution. We rely on a Metropolis-Hastings sampler for these parameters with a tuned normal random-walk distribution as the proposal density. We note that the determinant term $|B_o| = |I_n - \rho_o D|$, is calculated using the sparse matrix methods of Barry and Pace (1997) to compute and store tabled values for this determinant over a grid of q values for ρ_o in the interval $[\kappa_{\min}^{-1}, \kappa_{\max}^{-1}]$. This is done prior to beginning the MCMC sampling loop with table look-up used during sampling, allowing rapid evaluation of candidate values during sampling.

The joint posterior densities for σ_o^2, σ_d^2 take the form:

$$
\begin{aligned}
p(\sigma_o^2|\delta,\theta,\phi,\rho_o,\rho_d,\sigma_\varepsilon^2,y,Z) &\propto \pi(\theta|\rho_o,\sigma_o^2)\pi(\sigma_o^2) \\
&\propto (\sigma_o^2)^{-n/2}\exp\left(-\frac{1}{2\sigma_o^2}\theta' B_o' B_o \theta\right)(\sigma_o^2)^{-\frac{n}{2}+\nu_1+1}
\end{aligned}
$$

Which results in an inverse gamma distribution for the conditional posterior. A similar result applies to σ_d^2, with details provided in Appendix A.

$$
\sigma_o^2|\delta,\theta,\phi,\rho_o,\rho_d,\sigma_d^2,\sigma_\varepsilon^2 \sim IG(a,b)
$$

$$
\begin{aligned}
a &= (n/2) + \nu_1 \\
b &= \theta' B_o' B_o \theta + 2\nu_2 \qquad (19) \\
\sigma_d^2 | \delta, \theta, \phi, \rho_o, \rho_d, \sigma_o^2, \sigma_\varepsilon^2 &\sim IG(c, d) \\
c &= (n/2) + \nu_1 \\
d &= \phi' B_d' B_d \phi + 2\nu_2 \qquad (20)
\end{aligned}
$$

Finally, the conditional posterior distribution for the noise variance parameter σ_ε^2 takes the form of an inverse gamma distribution:

$$
\begin{aligned}
\sigma_\varepsilon^2 | \delta, \theta, \phi, \rho_o, \rho_d, \sigma_d^2, \sigma_o^2, y, Z &\sim IG(e, f) \\
e &= (n/2) + \nu_1 \\
f &= \nu' \nu + 2\nu_2 \qquad (21) \\
\nu &= y - Z\delta - V\theta - W\phi
\end{aligned}
$$

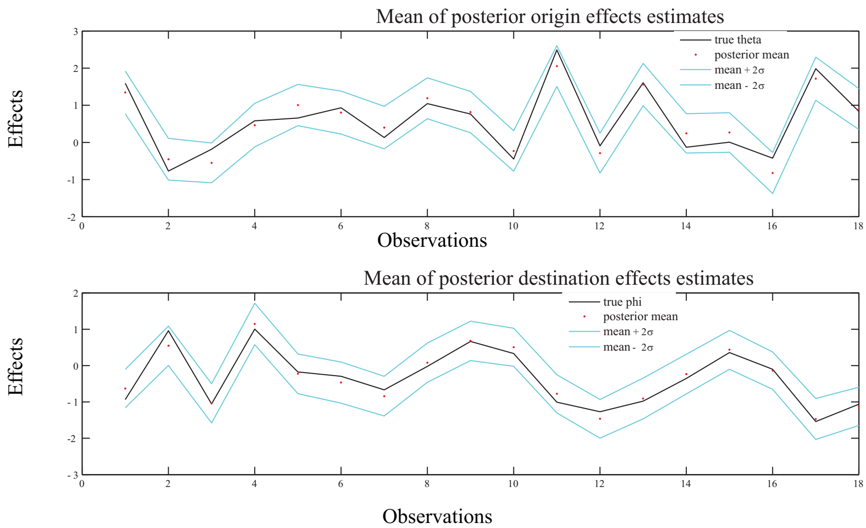
3 A data generated example

To illustrate our method, a sample of flows were generated using the model from (11) with the latitude-longitude coordinates from our sample of 18 Spanish regions used to produce a spatial weight matrix based on 5 nearest neighbors. The parameters of the model were set to: $\sigma_\varepsilon^2 = 1.5, \sigma_o^2 = 0.75, \sigma_d^2 = 0.5, \rho_o = 0.6, \rho_d = 0.7$. The parameters δ were associated with matrices X_o, X_d which reflected a random normal n by 2 matrix X repeated to form: $X_d = \iota_n \otimes X$ and $X_o = X \otimes \iota_n$. The parameters $\delta = (\delta_o, \delta_d)$ were set to: $\begin{pmatrix} 1 & -1 & -1 & 1 \end{pmatrix}$.[7]

[7] We exclude the intraregional model from the data generating process as well as estimation procedure.

The MCMC sampler was run to produce 5,500 draws with the first 2,500 discarded for burn-in of the sampler. Figure 1 shows the posterior means for the spatially structured regional origin and destination effects parameters along with a two standard deviation upper and lower limit. True effects parameters are also shown in the figure, where we see that the posterior means are within the two standard deviation limits.

Figure 1: Posterior estimates for origin/destination regional effects parameters



The posterior means and medians for the parameters δ along with their standard deviations and the true values are presented in Table 1. All of the estimates are within two standard deviations of the true values used to generate the data. The medians are near the means, indicating a symmetric posterior distribution.

An interesting contrast to the accurate estimates from the Bayesian hi-

Table 1: Posterior estimates for δ

Parameter	Posterior Mean	Posterior Median	Posterior standard deviation
δ_{o1}, truth = 1	0.9907	0.9912	0.0610
δ_{o2}, truth = -1	-1.0073	-1.0095	0.0745
δ_{d1}, truth = -1	-1.0760	-1.0752	0.0598
δ_{d2}, truth = 1	0.9167	0.9179	0.0740

Table 2: Least-squares estimates for δ

Parameter	OLS $\hat{\delta}$	Standard Deviation
δ_{o1}, truth = 1	1.0076	0.0861
δ_{o2}, truth = -1	-0.6739	0.1034
δ_{d1}, truth = -1	-1.1367	0.0861
δ_{d2}, truth = 1	0.7416	0.1034

erarchical spatial model are those from ordinary least-squares, shown in Table 2. Here, we see two of the four parameter estimates more than two standard deviations away from the true values used to generate the sample data. We note that the spatially structured regional effects vectors θ and ϕ were generated to have a mean of zero, so we would not expect bias in the least-squares estimates. Nevertheless, this appears to be the case here. The least-squares noise estimate for σ_ε^2 was 2.76, with the true value used to generate the data equal to 1.5.

Table 3 shows estimates for the parameters ρ_o, ρ_d, and Table 4 presents results for the variance parameters $\sigma_o^2, \sigma_d^2, \sigma_\varepsilon^2$. All of these posterior mean estimates are reasonably close to the true values used to generate the sample data.

Table 3: Posterior estimates for the spatial dependence parameters

Parameter	Posterior Mean	Posterior Median	Posterior standard deviation
ρ_d, truth = 0.6	0.5701	0.5834	0.2002
ρ_o, truth = 0.7	0.6290	0.6572	0.1986

Table 4: Posterior estimates for the variance parameters

Parameter	Posterior Mean	Posterior standard deviation
σ_d^2, truth = 0.5	0.5273	0.2041
σ_o^2, truth = 0.75	0.7546	0.2793
σ_ε^2, truth = 1.5	1.4107	0.1122

4 An application to commodity flows between Spanish regions

Here we present results from applying our methodology to the logged year 2002 commodity flows measured in Euros between 18 Spanish NUTS 2 regions.

4.1 The Data

In Spain, like in many other countries, there are no official data on the interregional trade flows. However, there are different estimates produced with alternative methodologies (Oliver et al, 2003; Llano, 2004). The data used in this article corresponds to recent estimates produced in the C-intereg project (www.c-intereg.es), by combining the most accurate data on Spanish transport flows of goods by transport modes (road, rail, ship and plane) with additional information regarding export price vectors, one per each region of origin, transport mode and type of product. The methodology also includes a process for debugging the original transport flows database, which allows the identification and reallocation of multi-modal transport flows and international transit flows hidden in the interregional flows. This procedure results in initial estimates of interregional trade flows in tons and euros. Finally, a process of harmonization is applied to produce final figures in tons and euros coherent with figures of total output from the Spanish Industrial Survey and the National Accounts. At each stage the methodology relies on the lowest level of disaggregation available.

The dependent variable was (logged) year 2002 flows within and between the 18 Spanish regions. Explanatory variables used in the model were characteristics of the origin and destination regions: the log of 1995 GDP, the log of year 1995 population density, the log of year 2002 kilometers of motorways in the region standardized by area of the region, foreign exports and imports measured in millions of Euros. In addition, the log of distance between regions constructed as a vectorized version of the OD distance matrix was added.

The population density and 1995 GDP were included to capture size, urban agglomeration and income effects. Foreign export and import trade variables were included as a proxy for openness to trade by the regions. We would expect that more foreign exports at the origin would lead to more interregional flows since this suggests firms have in place administrative structures to facilitate export trade. More foreign imports at the destination might also be positively related to interregional flows because this should correlate with intermediate-final requirements, as well as firms administrative structures to facilitate foreign imports. Since around 90%

Table 5: Least-squares gravity model estimates

Variable	Coefficient	t-statistic	t-probability
constant	-16.1051	-9.505	0.0000
D_GDP95	1.5018	17.262	0.0000
D_PopDensity	-0.1653	-2.238	0.0259
D_Motorways	0.2675	3.308	0.0010
D_Imports	0.3985	2.425	0.0158
D_Exports	-0.0392	-0.271	0.7864
O_GDP95	1.0589	12.172	0.0000
O_PopDensity	0.0237	0.321	0.7482
O_Motorways	0.0603	0.745	0.4564
O_Imports	-0.5913	-3.598	0.0003
O_Exports	0.6057	4.182	0.0000
log(Distance)	-1.2206	-14.995	0.0000

of Spanish interregional trade is moved by road, a motorways variable was included to capture regional infrastructure effects of these on commodity flows.

Least-squares estimates are presented in Table 5. Posterior estimates from the Bayesian hierarchical model in Table 6 are presented using $t-$statistics and associated probabilities constructed using the posterior means and standard deviations of the MCMC draws for ease of comparison with the least-squares estimates. The $r-$squared for least-squares was 0.8, suggesting a good fit to the data. All but three of the explanatory variables were significant at the 95% level or above. Origin and destination GDP in 1995 were both positive and near unity which indicates that interregional trade flows are roughly proportional to this measure of economic size of the regions, consistent with the underlying premise of the gravity model relationship. More foreign exports at the origin exert a positive impact on interregional exports as expected, whereas more foreign imports at the origins have a negative impact on the interregional deliveries. By contrast, more foreign imports at the destination implies more interregional imports, while foreign exports at the destinations have not significant effects on interregional imports. These results are coherent with previous findings (Llano et al, 2010) that suggest that regions with higher levels of foreign imports (exports) are associated with higher interregional imports (exports). Furthermore, as it was suggested in such previous analysis, the non-significant results for 'D_Exports' and the negative and significant for 'O_imports' are coherent with the inverse signs of the interregional and international trade balance obtained for some regions such as Castilla y León, Castilla-La Mancha, Comunidad-Valenciana, Cataluña or Madrid.

Motorways at the destination have a positive and significant impact on trade flows, while motorways at the origin are not significant. Finally, population density at the destination has a negative impact on interregional trade flows while that for origins is not significant. This suggest less interregional trade flows going to more densely populated regions where we would expect more intraregional flows to take place. Note that small (in surface) but dense populated and single-provincial regions such as Ceuta y Melilla, Madrid, Asturias or the Islands (Canarias and Baleares) are in this group. The insignificant coefficient for origin region population density is probably explained by the fact that some of the largest exporting regions to the rest of Spain are also big regions in terms of surface (Cataluña, País Vasco, Comunidad Valenciana, Castilla y León or Andalucía), whose population density are not among the largest in the country. Logged distance between regions has the expected negative impact on interregional trade flows.

Bayesian estimates are reasonably similar to those from least-squares,

Table 6: Bayesian Model Posterior estimates

Variable	Mean	$t-$statistic	$t-$probability
D_GDP95	1.3287	14.3966	0.0000
D_PopDensity	-0.5699	-7.0965	0.0000
D_Motorways	-0.1280	-1.3884	0.1660
D_Imports	0.2137	1.2046	0.2293
D_Exports	-0.0927	-0.5707	0.5686
O_GDP95	0.7356	8.0480	0.0000
O_PopDensity	-0.2437	-2.9809	0.0031
O_Motorways	-0.0399	-0.4220	0.6733
O_Imports	-0.8621	-4.7029	0.0000
O_Exports	0.6992	4.1297	0.0000
log(Distance)	-1.5071	-17.5146	0.0000

with a few exceptions that may be important. Origin and destination 1995 GDP was positive and significant with coefficients near one, but smaller in magnitude than the least-squares estimates. These estimates suggest that destination GDP is nearly twice as important in explaining interregional flows than origin GDP. Motorways are not significant at either the origin or destination, in contrast to least-squares where this coefficient was positive and significant for the destination. Population density at both the origin and destination were negative and significant, perhaps suggesting more intraregional flows for high population density regions and less interregional flows. Like in least-squares, origin region foreign imports exert a negative and significant impact on interregional flows. Origin region foreign exports are positive and significant, suggesting that openness to foreign trade is positively associated with more interregional exports to the rest of the country. Destination region foreign imports and exports are not significant in terms of their influence on interregional trade flows.

Table 7: Bayesian Model Posterior estimates

Parameter	Mean	Median	Standard Deviation	$t-$statistic
ρ_d	0.4883	0.5080	0.2208	2.2115
ρ_o	0.3386	0.3035	0.2239	1.5123
σ_d^2	0.3743		0.1841	2.0331
σ_o^2	0.8551		0.3681	2.3230
σ_ε^2	1.9850		0.1602	12.3908

A question of interest is whether the origin and destination individual effects exhibit spatial dependence. Table 7 shows the posterior estimates for the spatial dependence parameters. Posterior means, medians and standard deviations are presented along with $t-$statistics constructed using the standard deviations.

From the table we see that destination effects exhibit positive and significant spatial dependence, while origin effects exhibit positive but weak and possibly not significant dependence. These positive dependence estimates indicate that latent or unobserved variables are at work at destinations to create effects estimates that are similar to those from regions neighboring the destinations. In our view, this result is in line with the geographical structure of the Spanish interregional flows, where: 1) some regions, such as Cataluña, Madrid, Comunidad Valenciana or País Vasco, accumulate a large share of the outflows and are not surrounded by other regions with strong interregional exports; 2) while the main importing regions are clustered together in the western and southern part of the country. This spatial pattern will be clear in Figure 2 and Figure 4)

The posterior mean origin and destination effects estimates by regions are presented in tables 8 and 10 along with 0.05 and 0.95 credible intervals for these parameters constructed using the sample of draws. A positive effects parameter for the origin (destination) suggests that unobserved forces would lead to higher commodity flow levels at origin (destination) than predicted by the explanatory variables reflecting regional characteristics. Regions that have positive origin and destination effects parameters are those that exhibit higher levels of commodity flows not explained by their characteristics alone. These regions could have some natural advantage or benefit from spatial spillovers. In contrast, regions with negative effects parameters experience levels of commodity flows lower than would be expected given their regional characteristics. These regions could be experiencing a natural disadvantage or suffer from adverse spatial spillovers.

In terms of the origin effects parameters shown in Table 8, (also mapped in Figure 2 with accompanying legend in Figure 3) we see six negative and significant regional effects (Aragón, Balears, Castilla Y León, Castilla-La Mancha, Extremadura and Ceuta y Melilla) and no positive and significant effects. The map makes it clear that the negative and significant effects estimates are pointing to two island regions (Balears, Canarias) and two other peripheral regions located on the eastern border of Spain (Extremadura and Galicia). This is consistent with the notion that the regional effects estimates point to regions with physical/geographical disadvantages regarding origins of interregional trade flows. The negative and significant effects suggests that interregional flows originating from these regions are smaller than one would expect given regional characteristics and distance alone. Of course, our model does not include an explanatory variable indicating the friction that arises for the two island locations. Consistent with earlier comments regarding the relationship of our spatial autoregressive prior and Bonacich

Table 8: Bayesian Model Posterior Origin Effects Estimates

Region	0.05 HPDI Interval	Mean	0.95 HPDI Interval
ANDALUCIA	-1.4105	-0.7045	0.0015
ARAGON	-1.9017	-1.2078	-0.5140
ASTURIAS (PRINCIPADO DE)	-0.8699	-0.1944	0.4811
BALEARS (ILLES)	-3.0338	-2.3332	-1.6327
CANARIAS	-0.7961	-0.0276	0.7408
CANTABRIA	-0.8440	-0.1659	0.5122
CASTILLA Y LEON	-2.0980	-1.3756	-0.6532
CASTILLA-LA MANCHA	-2.0612	-1.3581	-0.6550
CATALUÑA	-0.8102	-0.0977	0.6148
COMUNIDAD VALENCIANA	-1.0164	-0.3483	0.3199
EXTREMADURA	-2.3244	-1.5877	-0.8511
GALICIA	-0.5959	0.0695	0.7348
MADRID (COMUNIDAD DE)	-0.8803	-0.1110	0.6583
MURCIA (REGION DE)	-0.9732	-0.2563	0.4605
NAVARRA (C. FORAL DE)	-1.0243	-0.3735	0.2774
PAIS VASCO	-0.4113	0.2529	0.9172
RIOJA (LA)	-1.3142	-0.5724	0.1694
CEUTA Y MELILLA	-2.3218	-1.4922	-0.6625

Table 9: Bayesian Model Posterior Destination Effects Estimates

Region	0.05 HPDI Interval	Mean	0.95 HPDI Interval
ANDALUCIA	0.0766	0.7187	1.3608
ARAGON	-0.5562	0.0653	0.6868
ASTURIAS (PRINCIPADO DE)	-0.3995	0.2126	0.8247
BALEARS (ILLES)	-1.2403	-0.6190	0.0024
CANARIAS	0.0614	0.7607	1.4600
CANTABRIA	-0.1628	0.4704	1.1037
CASTILLA Y LEON	-1.0215	-0.3497	0.3221
CASTILLA-LA MANCHA	-0.6487	-0.0163	0.6161
CATALUÑA	0.1153	0.7621	1.4088
COMUNIDAD VALENCIANA	0.1845	0.8258	1.4672
EXTREMADURA	-0.9758	-0.2921	0.3915
GALICIA	0.0091	0.6146	1.2201
MADRID (COMUNIDAD DE)	-0.2086	0.4573	1.1232
MURCIA (REGION DE)	-0.5898	0.0513	0.6923
NAVARRA (C. FORAL DE)	-0.6849	-0.0870	0.5110
PAIS VASCO	0.0345	0.6742	1.3138
RIOJA (LA)	-0.5113	0.1601	0.8316
CEUTA Y MELILLA	-0.7844	-0.0317	0.7211

Table 10: Bayesian Model Posterior Destination Effects Estimates

Region	0.05 HPDI Interval	Mean	0.95 HPDI Interval
ANDALUCIA	0.0766	0.7187	1.3608
ARAGON	-0.5562	0.0653	0.6868
ASTURIAS (PRINCIPADO DE)	-0.3995	0.2126	0.8247
BALEARS (ILLES)	-1.2403	-0.6190	0.0024
CANARIAS	0.0614	0.7607	1.4600
CANTABRIA	-0.1628	0.4704	1.1037
CASTILLA Y LEON	-1.0215	-0.3497	0.3221
CASTILLA-LA MANCHA	-0.6487	-0.0163	0.6161
CATALUÑA	0.1153	0.7621	1.4088
COMUNIDAD VALENCIANA	0.1845	0.8258	1.4672
EXTREMADURA	-0.9758	-0.2921	0.3915
GALICIA	0.0091	0.6146	1.2201
MADRID (COMUNIDAD DE)	-0.2086	0.4573	1.1232
MURCIA (REGION DE)	-0.5898	0.0513	0.6923
NAVARRA (C. FORAL DE)	-0.6849	-0.0870	0.5110
PAIS VASCO	0.0345	0.6742	1.3138
RIOJA (LA)	-0.5113	0.1601	0.8316
CEUTA Y MELILLA	-0.7844	-0.0317	0.7211

centrality, the map shows effects magnitudes that are near zero (not significantly different from zero) for regions located more centrally in Spain, and the negative effects for eastern border regions is also consistent with a lack of centrality.

The destination effects in Table 10 (also mapped in Figure 4 with accompanying legend in Figure 5) show only positive and significant regional effects for six regions (Andalucía, Canarias, Cataluña, Comunidad Valenciana, Galicia and País Vasco). However, the map shows that negative but not significant effects are indicated for the island region of Balears and the eastern border region Castilla Y León.

The positive effects observed in País Vasco, Cataluña and Comunidad Valenciana may be explained by the importance of their ports. Moreover, both País Vasco and Cataluña play a role as importing hubs from international markets, being surrounded by other important importing regions such as Cantabria, Navarra, Castilla y León or la Rioja (for the País Vasco), or Aragón and Comunidad Valenciana (for Cataluña). The sectoral and geographical specificities of these regions may explain the higher interregional imports of their neighbors, resulting in flows that would be higher than predicted by the destination characteristics included in the X_d matrix of explanatory variables. Of course, Cataluña is the home of Barcelona, and the main entree-door to the rest of Europe by road.

In addition to the two tables presenting origin and destination effects estimates a third table shows the posterior mean for the sum of both origin and destination effects along with 0.05 and 0.95 credible intervals. This was constructed using the sum of the draws for both origin and destination effects. Positive values for these combined parameter estimates provide us with an indication of which regions benefit from unobserved positive forces at work that lead to high levels of interregional commodity flows that originate and terminate in the region. Similarly, negative combined values for these parameters point to regions that suffer disadvantages leading to lower interregional commodity flow levels.

The combined effects in Table 11 shows six negative and no positive and significant effects parameters. Consistent with the discussion above, this final picture may capture the presence of possible disadvantages in those regions characterized by a relative backwardness in terms of income and accessibility that are located relatively far from the main axis of development (Mediterranean Arc; Ebro-Valley) and the most powerful regions in

Table 11: Bayesian Model Posterior Origin + Destination Effects Estimates

Region	0.05 HPDI Interval	Mean	0.95 HPDI Interval
ANDALUCIA	-0.9518	0.0142	0.9802
ARAGON	-2.1025	-1.1425	-0.1826
ASTURIAS (PRINCIPADO DE)	-0.8983	0.0182	0.9347
BALEARS (ILLES)	-3.9166	-2.9522	-1.9879
CANARIAS	-0.3131	0.7331	1.7792
CANTABRIA	-0.6434	0.3045	1.2525
CASTILLA Y LEON	-2.7152	-1.7254	-0.7355
CASTILLA-LA MANCHA	-2.3331	-1.3743	-0.4156
CATALUÑA	-0.2762	0.6644	1.6051
COMUNIDAD VALENCIANA	-0.4600	0.4776	1.4152
EXTREMADURA	-2.8785	-1.8799	-0.8813
GALICIA	-0.2394	0.6841	1.6075
MADRID (COMUNIDAD DE)	-0.6813	0.3463	1.3740
MURCIA (REGION DE)	-1.1733	-0.2050	0.7632
NAVARRA (C. FORAL DE)	-1.3549	-0.4604	0.4341
PAIS VASCO	-0.0037	0.9271	1.8579
RIOJA (LA)	-1.4412	-0.4122	0.6167
CEUTA Y MELILLA	-2.6561	-1.5238	-0.3916

terms of production and trade (Madrid, Cataluña, Andalucía, Comunidad Valenciana).

In conclusion, the spatial effects estimates based on our model of commodity flows aggregated across all commodity types seem plausible in that they were able to capture the most salient feature of Spanish interregional trade. This is seen in the concentration of flows within the eastern part of the country. In addition, the model demonstrated the significance of distance and the origin-destination characteristics of regions in explaining variation in the flows. (Nearly 80 percent of the variation in flows was explained by the model.)

Future work based on specific types of commodities as well as classification of goods into final or intermediate products may be important.

5 Conclusions

Gravity or spatial interaction models have traditionally relied on least-squares estimation methods, ignoring the issue of spatial dependence between interregional flows. We propose a modeling methodology that introduces spatially structured origin and destination region effects parameters. These parameters allow spatial heterogeneity to be modeled in such a way that regions treated as origins exhibit similar random effects levels to those of regions that neighbor the origins. A similar spatial structure is placed on random effects parameters for the regions viewed as destinations.

In contrast to typical conditional autoregressive spatial structure we rely on a spatial autoregressive prior to structure the random effects parameters. Our approach subsumes normally distributed random effects models as a special case when spatial dependence does not exist, so that $\rho_o = \rho_d = 0$. In addition, the effects parameter estimates can be used to diagnose the presence of positive or negative unobservable latent factors that influence interregional commodity flows.

In an application of the method to commodity flows among a sample of 18 Spanish regions, we found that least-squares estimates of the role played by regional characteristics differ greatly from those found by our Bayesian hierarchical spatial effects model. Introduction of the spatially structured random effects that account for heterogeneity across the regions appear to produce more efficient parameter estimates for the characteristics parameters.

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Appendix A: Details regarding the MCMC sampler

First, we show that the conditional posterior for δ takes the multivariate form presented in the text.

$$
\begin{aligned}
p(\delta|\theta, \phi, \rho_o, \rho_d, \sigma_o^2, \sigma_d^2, \sigma_\varepsilon^2) &\propto \pi(\delta) \cdot \exp\{-\frac{1}{2\sigma^2\varepsilon}(y - Z\delta - V\theta - W\phi)'(y - Z\delta - V\theta - W\phi)\} \\
&\propto \exp\{-\frac{1}{2\sigma^2\varepsilon}(y - Z\delta - V\theta - W\phi)'(y - Z\delta - V\theta - W\phi)\} \\
&\cdot \exp\{-\frac{1}{2\sigma^2\varepsilon}\delta' T^{-1}\delta\} \\
&\propto \exp\left(-\frac{1}{2\sigma^2\varepsilon}[\delta'(Z'Z + T^{-1})\delta - 2Z'(y - V\theta - W\phi)'\delta]\right) \\
&\propto \exp\left(-\frac{1}{2\sigma^2\varepsilon}[\delta - \Sigma_\delta^{-1}\mu_\delta]'\Sigma_\delta[\delta - \Sigma_\delta^{-1}\mu_\delta]\right)
\end{aligned}
$$

Where as reported in the text:

$$
\begin{aligned}
\mu_\delta &= \frac{1}{\sigma^2\varepsilon} Z'(y - V\theta - W\phi) \\
\Sigma_\delta &= \frac{1}{\sigma^2\varepsilon}(Z'Z + T^{-1})
\end{aligned}
$$

In this appendix we follow Smith and LeSage (2004) in deriving the conditional posterior for the spatial autoregressive effects parameters θ. They note that:

$$
\begin{aligned}
p(\theta|\ldots) &\propto \exp\left(-\frac{1}{2\sigma_\varepsilon^2}[V\theta - (y - Z\delta - W\phi)]'[V\theta - (y - Z\delta - W\phi)]\right) \\
&\cdot \exp\left(-\frac{1}{2\sigma_o^2}\theta' B_o' B_o \theta\right)
\end{aligned}
$$

$$
\begin{aligned}
&\propto \exp\left(-\frac{1}{2\sigma_\varepsilon^2}[\theta' V' V\theta - 2(y - Z\delta - W\phi)' V\theta + \theta'(\sigma_o^{-2} B_o' B_o)\theta]\right) \\
&= \exp\left(-\frac{1}{2\sigma_\varepsilon^2}[\theta'(\sigma_o^{-2} B_o' B_o + V'V)\theta - 2(y - Z\delta - W\phi)' V\theta]\right)
\end{aligned}
$$

from which it follows that:

$$
\begin{aligned}
\theta|\delta, \phi, \rho_o, \rho_d, \sigma_o^2, \sigma_d^2, \sigma_\varepsilon^2, y, Z &\sim N_n[\Sigma_\theta^{-1}\mu_\theta, \Sigma_\theta^{-1}] \\
\mu_\theta &= \sigma_\varepsilon^{-2} V'(y - Z\delta - W\phi) \\
\Sigma_\theta &= (\frac{1}{\sigma_o^2} B_o' B_o + \frac{1}{\sigma_\varepsilon^2} V'V)
\end{aligned} \tag{22}
$$

The conditional posteriors for σ_o^2, σ_d^2:

$$
\begin{aligned}
p(\sigma_o^2|\ldots) &\propto \pi(\theta|\rho_o, \sigma_o^2)\pi(\sigma_o^2) \\
&\propto (\sigma_o^2)^{-n/2} \exp\left(-\frac{1}{2\sigma_o^2}\theta' B_o' B_o \theta\right)(\sigma_o^2)^{\nu_1+1} \exp\left(-\frac{\nu_2}{\sigma_o^2}\right) \\
&\propto (\sigma_o^2)^{-(\frac{n}{2}+\nu_1+1)} \exp[-\theta' B_o' B_o \theta + \frac{2\nu_2}{2\sigma_o^2}]
\end{aligned}
$$

Which is proportional to the inverse gamma distribution reported in the text. A similar approach leads to $p(\sigma_d^2|\ldots)$, and the conditional posterior for σ_ε^2:

$$
\begin{aligned}
p(\sigma_\varepsilon^2|\ldots) &\propto (\sigma_\varepsilon^2)^{-n/2} \exp\left(-\frac{1}{2\sigma_\varepsilon^2} e' e\right)(\sigma_\varepsilon^2)^{\nu_1+1} \exp\left(-\frac{\nu_2}{\sigma_\varepsilon^2}\right) \\
&\propto (\sigma_\varepsilon^2)^{-(\frac{n}{2}+\nu_1+1)} \exp[-e'e + \frac{2\nu_2}{2\sigma_\varepsilon^2}] \\
e &= y - Z\delta - V\theta - W\phi
\end{aligned}
$$

Figure 2: Map of posterior estimates for origin effects parameters

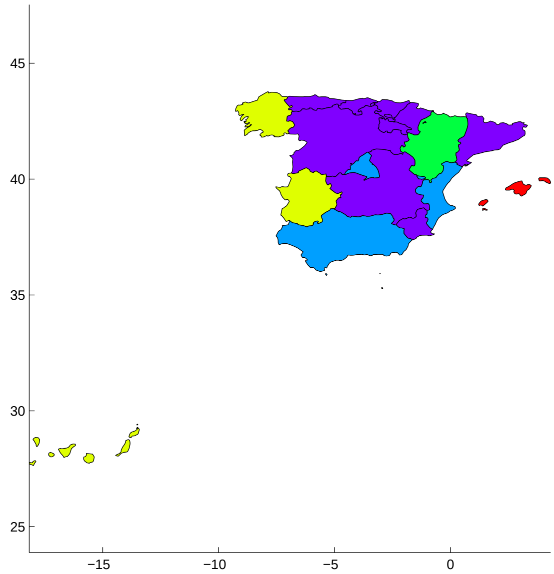

Figure 3: Histogram legend for origin effects parameters

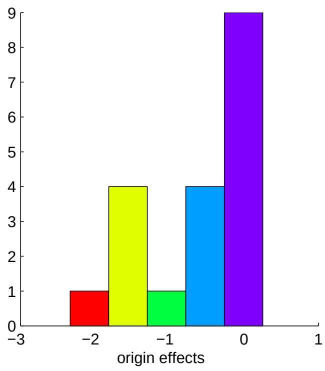

Figure 4: Map of posterior estimates for destination effects parameters

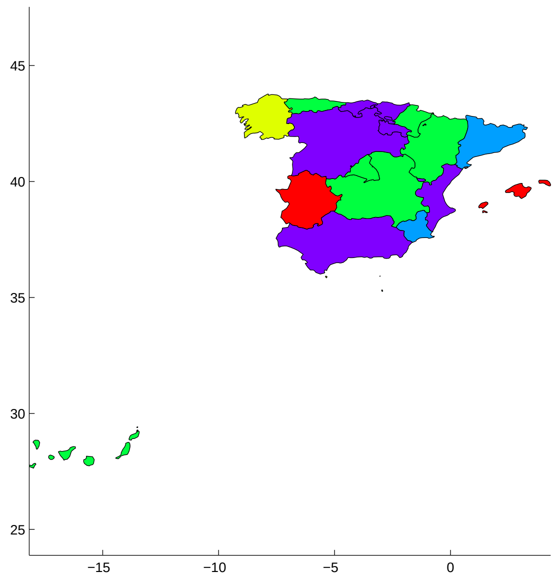

Figure 5: Histogram legend for destination effects parameters